

ERRATUM: POSITIVE TOPOLOGICAL ENTROPY FOR SEMI-RIEMANNIAN GEODESIC FLOWS

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ABSTRACT. In this note we correct a mistake in the proof of Theorem 1.1 of [1]. We also include an extra condition in the statement of Theorem 1.3 that is needed to hold.

The σ -tangent bundle $S_g^\sigma M$ of a semi-Riemannian manifold is not generally compact. The lack of compactness makes our argument insufficient to prove a Kupka-Smale theorem as it is wrongly claimed in Theorem 3.4 (Corollary 3.2 does not hold either). To prove Theorem 1.1 we only need to adapt these two results in section 3.4 as it is done in [2].

The corrected statement of Corollary 3.2 is the following:

Corollary 3.2. *There exists a residual set $\mathcal{O} \subset \mathcal{SR}_\nu^r(M)$ such that for $g \in \mathcal{O}$ and any $\tau > 0$, the set of periodic orbits of φ_g intersecting a compact subset $K \subset S_g^\sigma M$ with period $\leq \tau$ is finite.*

Proof. Suppose there are infinite periodic orbits with period $\leq \tau$ intersecting a compact subset $K \subset S_g^\sigma M$. Take a point in each of these periodic orbits and belonging also to K . Since $K \subset S_g^\sigma M$ is compact, there is an accumulation point. This point sits in a periodic orbit because the orbits of the approximating periodic orbits are all bounded by τ . Therefore, there is a non isolated periodic orbit which is degenerate. Theorem 3.1 completes the proof. \square

The corrected statement of Theorem 3.4, and the necessary adaptation of its proof to the non-compact setting is the following:

Theorem 3.4. *For a C^r -residual set $\mathcal{R} \subset \mathcal{SR}_\nu^r(M)$, $r \geq 2$ or $r = \infty$, all heteroclinic points of hyperbolic periodic orbits intersecting any compact set $K \subset S_g^\sigma M$, are transversal.*

Consider the exhaustion of $S_g^\sigma M$:

$$K_1 \subset K_2 \subset \cdots \subset K_i \subset K_{i+1} \subset \cdots \subset S_g^\sigma M,$$

where each K_i is compact with nonempty interior, and satisfies $\text{int}(K_i) \subset K_{i+1}$, $i \geq 1$. Now we consider two sets of metrics:

- \mathcal{K}_{N,K_i}^r is the subset of metrics $g \in \mathcal{SR}_\nu^r(M)$ such that every periodic orbit of φ_g^t intersecting K_i and displaying period $\leq N$ are hyperbolic or q -elliptic.
- $\tilde{\mathcal{K}}_{N,K_i}^r$ is the subset of metrics $g \in \mathcal{K}_{N,K_i}^r$ such that the stable and unstable manifolds of the hyperbolic periodic orbits intersecting K_i with period $\leq N$ are transversal.

By Corollary 3.2, generically there are only a finite number of those hyperbolic periodic points with period $\leq N$ and intersecting K_i .

Since the stable and unstable manifolds of a periodic orbit depend continuously on compact parts in the C^1 topology, we conclude that $\tilde{\mathcal{K}}_{N,K_i}^r$ is an open subset of $\mathcal{SR}_\nu^r(M)$. If we prove that $\tilde{\mathcal{K}}_{N,K_i}^r$ is also dense, then the residual subset of Theorem 3.4 is defined by

$$\mathcal{R} = \bigcap_{N=1}^{\infty} \bigcap_{i=1}^{\infty} \tilde{\mathcal{K}}_{N,K_i}^r.$$

The proof that $\tilde{\mathcal{K}}_{N,K_i}^r$ is dense in $\mathcal{SR}_\nu^r(M)$ follows from Lemma 3.5, which completes the proof of Theorem 3.4.

Since compactness is essential in the proof of Theorem 1.3, in particular for the use of the Smale's spectral decomposition theorem, we need to adapt condition (2) of the definition of \mathcal{H} in the following way: "there are infinitely many non-lightlike periodic orbits with infinitely many different periods in a compact subset".

In page 3990 where it is written "Moreover, as M is compact so is $S_g^\sigma M$." should be "Moreover, even if M is compact, $S_g^\sigma M$ might not be compact."

In page 3990 the paragraph "It is widely known that the [...] does not depend on the metric." should be removed.

In page 3991 where it is written "Consider any distance d on $S_g^\sigma(M)$ " should be "Consider the distance d on $S_g^\sigma(M)$ ".

REFERENCES

- [1] M. Bessa, J. Lopes Dias, P. Matias and M. J. Torres, *Positive topological entropy for semi-Riemannian geodesic flows*, Proc. Amer. Math. Soc. 153 (2025), 3987–3998.
- [2] M. M. Peixoto, *On an Approximation Theorem of Kupka and Smale*, J. Differ. Equ. 3 (1967), 214–227.

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